

Review Calculus BC

①

$$f(x) = \frac{1}{x+1} \text{ around } x=1$$

$$\text{Force into } \frac{a}{1-r} : \frac{1}{1+(x-1)+1} = \frac{1}{2+(x-1)} = \frac{1}{2-[-(x-1)]}$$

$$= \frac{1/2}{1 - \frac{1}{2}[-(x-1)]} = \frac{1/2}{1 - [(-1/2)(x-1)]} \quad a = 1/2 \quad r = (-1/2)(x-1)$$

$$\sum \frac{1}{2} \left(-\frac{1}{2}\right)^n (x-1)^n \text{ OR } \sum_{n=0}^{\infty} (-1)^n \left(\frac{1}{2}\right) \left(\frac{x-1}{2}\right)^n$$

b) radius of Convergence :

$$R = 2$$

$$\left| \frac{x-1}{2} \right| < 1$$

$$|x-1| < 2$$

$$-2 < x-1 < 2$$

$$-1 < x < 3$$

Interval of Convergence

Geometric Endpoints Do not work

$$c) \frac{1}{2} - \frac{x-1}{4} + \frac{(x-1)^2}{8} - \frac{(x-1)^3}{16} \bigg|_{x=.5} =$$

$$\textcircled{2} \sum \frac{nx^n}{2^n} \quad \lim_{n \rightarrow \infty} \left| \frac{(n+1)x^{n+1}}{2^{n+1}} \cdot \frac{2^n}{nx^n} \right| = \left| \frac{x}{2} \right| \lim_{n \rightarrow \infty} \left(\frac{n+1}{n} \right) = \left| \frac{x}{2} \right| < 1$$

$$\left| \frac{x}{2} \right| < 1 \Rightarrow |x| < 2 \quad -2 < x < 2$$

$$\text{Endpoints: } x = -2 \quad \sum \frac{n(-2)^n}{2^n} = \sum \frac{n(-1)^n \cdot 2^n}{2^n} = (-1)^n \cdot n \quad \text{Diverges } n^{\text{th}} \text{ term}$$

$$x = 2 \quad \sum n \quad \text{Diverges } n^{\text{th}} \text{ term}$$

(2b on back page)

③ a) $L(x) = -1 + 2x$

b) $P_2(x) = -1 + 2x - \frac{3x^2}{2!}$

c) $P_3(x) = -1 + 2x - \frac{3x^2}{2!} + \frac{4x^3}{3!}$ at $x=.1 = -.106\bar{3}$

④ $\sum \frac{\pi^n}{e^{2n}} = \frac{\pi^n}{(e^2)^n} = \sum_{n=0}^{\infty} \left(\frac{\pi}{e^2}\right)^n$ Sum = $\frac{1}{1 - (\frac{\pi}{e^2})} = 1.7396$

⑤ $f(x) = \sum 2 \left(\frac{x+2}{3}\right)^n$ Geometric so $a=2$ $r = \frac{x+2}{3}$

$-1 < \frac{x+2}{3} < 1$ $\boxed{-3 < x+2 < 3}$ $\boxed{-5 < x < 1}$ Endpoints Do not work because series is geometric

b) $\frac{2}{1 - \frac{(x+2)}{3}} = \frac{2}{\frac{3-x-2}{3}} = \frac{2}{\frac{1-x}{3}} = \frac{6}{1-x}$

6. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \sum_{n=0}^{\infty} \frac{x^n}{n!}$ $\leftarrow n=0$

$e^x - 1 = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \sum_{n=1}^{\infty} \frac{x^n}{n!}$ $\leftarrow n=1$

$\frac{e^x - 1}{x} = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \dots + \sum_{n=1}^{\infty} \frac{x^{n-1}}{n!}$

$\int \frac{e^x - 1}{x} = x + \frac{x^2}{2 \cdot 2!} + \frac{x^3}{3 \cdot 3!} + \dots + \sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!}$

$$\textcircled{7} \sum \frac{(-1)^n}{\sqrt[3]{n+2n}}$$

To converge absolutely ... $\sum \frac{1}{\sqrt[3]{n+2n}}$

but this series diverges

must converge.

by the Integral Test

so it is conditionally convergent.

$$2b) \text{ when } x = -1 \quad f(x) = \sum_{n=0}^{\infty} n \left(-\frac{1}{2}\right)^n$$

$$f(x) = -\frac{1}{2} + \frac{2}{4} - \frac{3}{8} + \frac{4}{16} - \frac{5}{2^5} + \frac{6}{2^6} - \frac{7}{2^7} + \frac{8}{2^8} - \frac{9}{2^9}$$

the next term gives the maximum error
for an alternating Series

$$10^{\text{th}} \text{ term is } \frac{10}{2^{10}} < .01$$